## CREEP OF HARDENING MATERIALS WITH DIFFERENT

## PROPERTIES IN TENSION AND COMPRESSION

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It was pointed out in [1] that light alloys usually had different creep properties in tension and compression. An attempt was also made in [1] to describe the creep processes of such materials for the simplest case, in which the steady creep velocity obeyed a power relationship with respect to stress, the power index remaining constant. The possibility of describing the creep of nonhardening materials of a more general anisotropy was indicated in [2]; different relationships were employed for the stress subspace according to whether the linear invariant of the stress tensor obeyed the relations $\sigma_{i i}>0$ or $\sigma_{i i}<0$. Each of these relationships incorporated characteristics of the material determined solely from tension or solely from compression experiments, respectively. In the present investigation, we shall consider creep experiments carried out on titanium alloys at room temperature, and shall show that an analogous method of describing the properties (in accordance with the sign of the first invariant of the stress tensor) is also applicable to the case of hardening materials with different properties in tension and compression.

1. As original material we took a sheet of titanium alloy $20-\mathrm{mm}$ thick. The direction of the greatest dimension of the sheet we shall subsequently call the longitudinal direction. The blocks used for preparing the samples were cut in the longitudinal and transverse directions, and also at $45^{\circ}$ to the former (diagonal direction). Depending on the precise nature and program of the experiment, samples of a variety of shapes were prepared from these blocks; the effect of the type of machining employed and the mode of subsequent trimming was first studied so as to be able to choose and specify a strict set of working conditions. The samples were not heat-treated after preparation.

In order to plot the $\sigma^{-} \varepsilon_{0}$ diagrams, we made plane samples $5 \times 10 \mathrm{~mm}^{2}$ in size with a working length of 100 mm . In order to plot the analogous diagrams in compression, we made cylindrical samples 12 mm in diameter with a working length of 40 mm . Figure 1 illustrates the $\sigma-\varepsilon_{0}$ diagrams. Here and subsequently we shall express $\sigma_{i j}$ as a dimensionless quantity - the ratio of the current stress to the tensile yield stress $\sigma_{0.2}$. In the diagram the light points represent the experimental data for the elongation of the samples in the longitudinal direction, the crosses and triangles for the transverse and diagonal direction respectively. The dark points represent data corresponding to the compression of samples cut in the longitudinal direction. It follows from these diagrams that the material is practically isotropic in the sense of instantaneous elastic-plastic properties, and has the same properties in tension and compression.

The picture changes completely when creep processes are considered. Creep experiments were carried out on samples of rectangular section ( $10 \times 20 \mathrm{~mm}^{2}$ ) with a working length of 160 mm . During the experiments, the axial elongation of the sample and the change in width and thickness at the middle of the working section were measured. A number of experiments were carried out on samples of the same type as those used in plotting the $\sigma-\varepsilon_{0}$ diagrams, and also on tubular samples. The differences in the creep diagrams obtained for samples of different geometrical shapes (cross sections) were negligible.

Compressive creep experiments were carried out on cyclindrical samples of the same size as those used for the $\sigma-\varepsilon_{0}$ diagrams. The axial elongation and the changes in diameters in two perpedicular directions were measured by means of micron dial-type gages; one of the directions corresponded to the normal to the sheet, as in the tensile experiments.

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Fig. 1


Fig. 2

Figure 2 shows some typical creep diagrams. Here the light circles indicate the axial strains in tension and the dark circles the axial strains in compression; the crosses represent the transverse strains normal to the sheet. Figure 2a gives the results obtained with samples taken in the longitudinal direction, Fig. 2b those of samples taken in the transverse direction. The stresses at which the experiments were conducted are indicated on the diagrams. The creep diagrams for the diagonal samples are analogous.

On considering these diagrams, we see that, in the creep sense, the material is very anisotropic, and possesses different properties in tension and compression. The sum of the two transverse strains (in modulus) is approximately equal to the corresponding modulus of the axial strain, i.e., in the course of creep, the material behaves as an almost incompressible medium. It is interesting to note that, in all the tensile experiments, the modulus of the sum of the transverse strains never exceeded the axial strain, whereas in the compression tests the modulus of the corresponding deformation never exceeded the sum of the transverse strains.

Several experiments lasted more than 1000 h , but the ratio between the transverse strains remained unaltered, i.e., in the course of creep the initial anisotropy of the material underwent no appreciable changes. It follows from the curves illustrated that, in the creep sense, the weakest direction is the normal to the sheet and the strongest the transverse direction; the diagonal direction is intermediate, this being valid in respect of both tension and compression.

The strength properties of the material (in the creep sense) are very similar for the longitudinal direction of the sheet and the direction along the normal to the latter (Fig. 2b), i.e., samples cut from the transverse direction of the sheet may be regarded as transversally isotropic. This latter circumstance enabled us to prepare tubular samples with their axes lying along the transverse direction of the sheet, and to regard these samples as having creep properties of axial symmetry; this was particulary important in conducting experiments involving simultaneous tensile stresses and torque.

The creep deformation associated with both tension and compression was approximated by the relation

$$
\begin{equation*}
\varepsilon^{\alpha} d \varepsilon=B \sigma^{n} d t \tag{1.1}
\end{equation*}
$$

Figure 3 shows the experimental results in coordinates of $\log \varepsilon-\log t$ for stresses of $\sigma=0.826$ in tensile experiments and $\sigma=0.930$ (line 1) and $\sigma=0.870$ (lines 2 and 3 ) in compressive experiments, and in coordinates of $\log \varepsilon-\log \sigma$ for $t=100 \mathrm{~h}$. Here the light points represent tensile and the dark points compressive data, the figures on the corresponding lines having the following meanings: 1) Results obtained for samples taken in the longitudinal direction; 2) transverse direction; 3) diagonal direction. Using the data of Fig. 3 we determined the characteristics of Eq. (1.1):
tension

$$
\begin{gather*}
\alpha=2.3, \quad n=64  \tag{1.2}\\
B_{1}=30 \cdot 10^{-6}, \quad B_{2}=0.27 \cdot 10^{-6}, \quad B_{3}=2.35 \cdot 10^{-6} \quad[1 / \mathrm{h}]
\end{gather*}
$$


compression

$$
\begin{gather*}
\alpha^{*}=6.69, \quad n^{*}=103 \\
B_{1}^{*}=572 \cdot 10^{-18}, \quad B_{2}^{*}=0.021 \cdot 10^{-18}  \tag{1.3}\\
B_{3}{ }^{*}=120 \cdot 10^{-18}[1 / \mathrm{h}]
\end{gather*}
$$

Here the numbers 1,2 , and 3 , respectively, relate to the longitudinal, transverse, and diagonal directions.

The computed curves plotted from (1.1) with the characteristics of (1.2) and (1.3) are indicated by the lines in Fig. 2. The agreement between the computed curves and the experimental data is on the whole quite satisfactory.
2. In order to describe creep processes in such media in a complex state of stress, we shall assume (as in [2]) that there exist two potential functions describing the creep strain velocities $\eta_{i j}$ in the form

$$
\begin{equation*}
\Phi_{1}=\left(\frac{\sqrt{S}}{N}\right)^{\alpha} T_{1}^{1 / 2(n+1)}, \quad \eta_{i j}=\frac{\partial \Phi_{1}}{\partial \sigma_{i j}} \tag{2.1}
\end{equation*}
$$

for the stress range $\sigma_{\mathrm{ii}}>0$

$$
\begin{equation*}
\Phi_{2}=\left(\frac{\sqrt{S}}{N}\right)^{\alpha^{*}} T_{2}^{1 / 2\left(n^{*}+1\right)}, \quad \eta_{i j}=\frac{\partial \Phi_{2}}{\partial \sigma_{i j}} \tag{2.2}
\end{equation*}
$$

for the stress range $\sigma_{i i}<0$.
Here $N=\sigma_{i j} \varepsilon_{i j}$ for $\sigma_{i j} \varepsilon_{i j} \geq 0$, and we shall consider $N$ as being identically equal to zero for $\sigma_{i j} \varepsilon_{i j}<0$

$$
\begin{gathered}
S=3 \sigma_{i j}{ }^{\circ} \sigma_{i j}{ }^{\circ}, \quad \sigma_{i j}{ }^{0}=\sigma_{i j}-1 / 3 \delta_{i j} \sigma_{k k} \\
T_{1}=A_{11}\left(\sigma_{22}-\sigma_{33}\right)^{2}+A_{22}\left(\sigma_{33}-\sigma_{11}\right)^{2}+A_{33}\left(\sigma_{11}-\sigma_{22}\right)^{2}+2 A_{12} \sigma_{12}{ }^{2}+2 A_{23} \sigma_{23}{ }^{2}+2 A_{31 \sigma_{31}{ }^{2}}
\end{gathered}
$$

The expression for $T_{2}$ has an analogous form, with coefficients $A_{i j}^{*}$, the material being consider orthotropic and the coordinate system coinciding with the principal axes of anisotropy.

If we successively put $\sigma_{11}, \sigma_{22}$, and $\sigma_{33}$ as the only nonzero components in (2.1) and compare with (1.1), after substituting the corresponding characteristics of (1.2) we find $A_{11}, A_{22}, A_{33}$. Expressing the stress components $\sigma_{i j}$ acting on an area making an angle of $45^{\circ}$ with the direction $\mathrm{X}_{1}$ in the $\mathrm{X}_{1} \mathrm{X}_{2}$ plane, after some
simple transformations [3] and comparison with (1.1), allowing for (1.2), we find $A_{12}$. The coefficients $A_{23}$ and $\mathrm{A}_{31}$ may be found analogously from experimental data relating to the diagonal directions in the $\mathrm{X}_{2} \mathrm{X}_{3}$ and $\mathrm{X}_{3} \mathrm{X}_{1}$ planes. The method of finding $\mathrm{A}_{\mathrm{ij}} *$ from (2.2), allowing for (1.1) and (1.3), is analogus.

The validity of Eqs. (2.1), with coefficients $A_{i j}$ found from the characteristics of (1.2), was verified by creep experiments with tubular samples subjected to simultaneous tensile stress and torque.

The original blocks used for making the samples were cut from the transverse direction of the sheet. The tubular samples were machined with external and internal diameters of 17 and 15 mm , respectively, and a working length of 50 mm . The axial stress was determined as the ratio of the tensile stress to the cross-sectional area of the sample, the shear stress as the ratio of the torque to the cross-sectional area, multiplied by the average ring radius. The state of stress was regarded as uniform and constant during the creep process.

In describing these experiments, the quadratic forms in (2.1) take the form

$$
\begin{equation*}
S=2 \sigma^{2}+6 \tau^{2}, \quad T_{1}=\left(A_{11}+A_{33}\right) \sigma^{2}+2 A_{21} \tau^{2} \tag{2.3}
\end{equation*}
$$

Here we allow for the fact that the material has axial symmetry relative to the $X_{2}$ axis, i.e., $A_{21}=A_{23}$, and we introduce the shear stress $\tau$ in accordance with the relations $\sigma_{21}=-\tau \sin \varphi, \sigma_{23}=\tau \cos \varphi$, where the angle $X_{1}$ is reckoned from the $X_{1}$ axis in the $X_{1} X_{3}$ plane. From experimental data in uniaxial tension (1.2) we find

$$
A_{11}+A_{38}=0.5389, \quad 2 A_{21}=1.6807
$$

From (2.1) and (2.3) we obtain

$$
\begin{gather*}
(\sigma \varepsilon+\tau \tau)^{\alpha+1}=(\alpha+1)(n+1)\left(2 \sigma^{2}+6 \tau^{2}\right)^{1 / 2 \alpha}\left[\left(A_{11}+A_{33}\right) \sigma^{2}+2 A_{21} \tau^{2}\right]^{1 / 2(n+1)} t \\
\frac{\Delta \varepsilon}{\Delta \gamma}=\frac{C_{1} \sigma-C_{3 \varepsilon}}{C_{2} 3 \tau-C_{3} \gamma} \tag{2,4}
\end{gather*}
$$

Fiere

$$
\begin{gather*}
C_{1}=\frac{2 \alpha\left[\left(A_{11}+A_{33}\right) \sigma^{2}+2 A_{21} \tau^{2}\right]}{2 \sigma^{2}+6 \tau^{2}} \neq(n+1)\left(A_{11}+A_{33}\right) \\
C_{2}=\frac{2 \alpha\left[\left(A_{11}+A_{33}\right) \sigma^{2}+2 A_{21} \tau^{2}\right]}{2 \sigma^{2}+6 \tau^{2}}+\frac{(n+1) 2 A_{21}}{3} \\
C_{3}=\frac{\alpha\left[\left(A_{11}+A_{33}\right) \sigma^{2}+2 A_{21} \tau^{2}\right]}{\sigma \varepsilon+\tau \gamma} \tag{2.5}
\end{gather*}
$$

Expanding the indeterminacy in the second expression of (2.4) which occurs at the instant of loading when $\varepsilon=\gamma=0$, we find

$$
\Delta \varepsilon / \Delta \gamma=C_{1} \sigma / C_{2} 3 \tau
$$

If the stresses remain constant, then so does the latter ratio.
All the experimental results were compared with the curves computed from (2.4). In Fig. 4 the light points represent the axial deformations and the dark points the shear strains $\gamma$. The continuous lines indicate the computed curves for the axial strains $\varepsilon$ and the broken lines indicate the computed curves for the shear strains $\gamma$. The stresses $\sigma$ and $\tau$ at which the experiments were conducted are indicated in the diagrams. For pure torque on the sample, a slight axial elongation occurred; this is reflected in the corresponding diagram. In the same figure, the heavy broken line gives the calculated curve based on the characteristic $2 \mathrm{~A}_{21} *=1,2835$ obtained from experimental results for pure axial compression (1.3). As we should expect, the latter curve passes below the analogus curve based on the characteristics corresponding to pure tension.

We may conclude from the results presented (Fig. 4) that, in order to describe the creep of materials with different properties in tension and compression, we may justifiably use the method based on the introduction of different relationships for the stress ranges $\sigma_{\mathrm{ii}}>0$ and $\sigma_{\mathrm{ii}}<0$ respectively. For states of stress
having $\sigma_{\mathrm{ii}} \approx 0$, in which both characteristics of the material exert an influence, the method here proposed is inapplicable.

## LITERATURE CITED

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